HYPERSONIC FLOWS AROUND THIN BLUNTED POWER-LAW SHAPES

(GIPERZVUKOVOE OBTEKANIE TONKIKH PRITUPLENNYKH TEL STEPENNOI FORMY)

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Chernyi [1,2] developed the laws of similitude for flows around thin blunted cones and wedges. In the present note, it is shown that within the same framework analogous laws are valid for thin blunted power-law shapes, both two-dimensional ($\nu = 0$) and axisymmetric.

Let the form of the bodies be described by

$$r = \alpha L X^n \qquad (\alpha \ll 1, \;\; r \gg r_0, \;\; n = \text{const})$$

Here L represents some linear dimension, $x = LX$, the axial coordinate with origin at the nose, and r_0, the radius of the blunted nose. According to the law of similitude for flows around thin blunted bodies [3] the solution of the problem depends on the parameters

$$M\alpha, \qquad K = \frac{1}{2} c_x r_0^{\,1+\nu} \alpha^{-(3+\nu)} L^{-(1+\nu)}, \qquad \gamma, \; n$$

Here M is the Mach number of the oncoming stream, c_x the nose drag coefficient, and γ the adiabatic parameter. The problem lacks a characteristic length so that L may be removed from the system of determining parameters, as in [1] and [2], for instance by setting $K = 1$. Then the solution, specifically the quantities p and R ($\alpha^2 \rho_\infty U_\infty^{\,2} p$ is the pressure on body; $\alpha L R$ is the shape of shock wave; ρ_∞ is the free-stream pressure; U_∞ is the free-stream velocity) will be functions of the variable

$$X = \frac{x}{r_0} \alpha^{(3+\nu)/(1+\nu)} \left(\frac{2}{c_x}\right)^{1/(1+\nu)}$$

In order to obtain qualitative results let us apply an approximate gross method, corresponding to that in [1] and [2]. Let us use the law of plane sections* and the integrals of the laws of motion, and in the evaluation of the kinetic energy and of the impulse of the gas, let us assume that the whole mass of the disturbed gas is concentrated immediately downstream of the shock wave and that the velocity of this mass (normal to the axis of the body) is identical with the velocity of the gas just at the shock wave**. In the evaluation of the internal energy of the disturbed gas, let us assume that in the high-entropy low-density layer the pressure is constant at each section X = const and equal to the pressure on the body at that section. In the highly compressed regions of the gas, we shall take the pressure and density to be equal to their respective values just behind the shock wave. The assumption concerning the density is here necessary for the evaluation of the thickness of the region of the highly compressed gas.

With these assumptions the energy and impulse equations (per unit angle between meridional sections in the axisymmetric case) become

$$\left(\frac{2}{\gamma+1}\right)^2 R'^2 R^{1+\nu} + \frac{p}{\gamma-1}\left(\frac{2}{\gamma+1} R^{1+\nu} - X^{n(1+\nu)}\right) = 1 + 2^\nu n \int_0^X p X^{n(1+\nu)-1} dX$$

$$\frac{2}{\gamma+1} R^{1+\nu} R' = 2^\nu \int_0^X p R^\nu dX \qquad (1)$$

Here, the assumption $M\alpha \gg 1$ was made, for the sake of simplicity.

As $X \to 0$ the solution of Equations (1) take the form (2), and is an approximation to the blast-wave solution [4]

$$p = \varkappa_\nu X^{-2(1+\nu)/(3+\nu)}, \qquad R = \chi_\nu X^{2/(\nu+3)}$$

$$\varkappa_0 = \frac{4}{9(\gamma+1)} \chi_0^2, \qquad \chi_0 = \left[\frac{9}{2}\left(\frac{\gamma+1}{2}\right)^2 \frac{\gamma-1}{2\gamma-1}\right]^{1/3}$$

$$\varkappa_1 = \frac{1}{2}\sqrt{\frac{\gamma-1}{2(2\gamma-1)}}, \qquad \chi_1 = 2\sqrt{(\gamma+1)\varkappa_1} \qquad (2)$$

For γ = 1.4, the Equations (2) yield values of pressure which exceed the exact blast-wave results [4] by 10% for ν = 0 and by 20% for ν = 1. (These differences are two and a half times smaller than those in [1,2].)

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* The equivalence principle of Hayes.

** For details of such procedure see [3], English translation pp. 218-220.

As $X \to \infty$, the solution takes the form

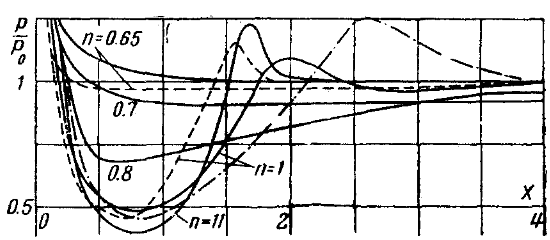
$$p = p_0 = CX^{2(n-1)}, \qquad R = R_0 = BX^n \tag{3}$$

Here, the following notation is used

$$C = \frac{2^{1-\nu} nm}{\gamma + 1} B^2, \qquad B^{1+\nu} = \frac{(\gamma^2 - 1)\, m}{2\,[2^\nu n\,(\gamma - 1) + m]} \left(\frac{2^\nu n}{m + n - 1} + \frac{1}{\gamma - 1} \right)$$

$$m = 2n + n\nu - 1$$

For the axisymmetric case with $\gamma = 1.4$ the Expression (3) gives values of pressure which differ from those of the exact solution [5] by not more than 5% in the range $0.65 < n < 1$. (In fact, the approximate solutions match the exact ones for $n \approx 1$ and $n \approx 0.73$.)



Hence, the Equations (1) are sufficiently accurate to be used for qualitative comparison of the effects of various parameters.

The results for p/p_0 of numerical integrations of the system (1) for $\nu = 1$ and for various values of n are displayed in the adjoining figure: solid lines designate the case $\gamma = 1.4$ and dotted lines $\gamma = 1.2$. For comparison, the approximate solution 2 for $\nu = n = 1$ and $\gamma = 1.4$ is shown as the line of points and dashes.

It is clear that a decrease in γ leads to a shortening of the region of influence due to blunting. As the parameter n decreases, the curves of p/p_0 lose their oscillatory character and become almost monotonic as n approaches 0.65. Let C_x and C_{x_0} represent the drag coefficients of the blunted and of the original body up to a given X. For $\gamma = 1.4$ and $\nu = 1$, the ratio C_x/C_{x_0} does not have a minimum for $n \leqslant 1$ and approaches unity monotonically from above as X increases indefinitely. Hence, the drag of a blunted power-law body with $n \leqslant 1$ is always larger than the drag of the sharp body no matter what the length to bluntness ratio x/r_0. However, for $n = 1.1$, the drag ratio C_x/C_{x_0} has a minimal value of 0.9 when $x = 1.5$. We note that Chernyi [2] found such a minimum when $n = 1$.

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